



UNIVERSITY
OF MANITOBA

DEPARTMENT OF ELECTRICAL AND COMPUTER
ENGINEERING

24.781 COMPUTATIONAL ELECTROMAGNETICS

Take Home Final Exam

December 19, 2000

Due Date: 11:00 a.m., Wednesday, December 22, 2000

General Instructions:

- 1) This is a **take home** exam.
- 2) There are **5** questions in total.
- 3) Answer **all** questions as completely as possible.
- 4) All questions have equal weighting.
- 5) Print clearly.
- 6) Clearly indicate all the assumptions you've made in your solutions.

- 1) The linear system of equations

$$\begin{bmatrix} 1 & -a \\ -a & 1 \end{bmatrix} \mathbf{x} = \mathbf{b}$$

where a is real, can under certain conditions be solved by the iterative method

$$\begin{bmatrix} 1 & 0 \\ -\omega a & 1 \end{bmatrix} \mathbf{x}^{(k+1)} = \begin{bmatrix} 1-\omega & \omega a \\ 0 & 1-\omega \end{bmatrix} \mathbf{x}^{(k)} + \omega \mathbf{b} .$$

(a) For which values of a is the method convergent for $\omega = 1$.

(b) For $a = 0.5$, find the value of $\omega \in \{0.8, 0.9, 1.0, 1.1, 1.2, 1.3\}$ which minimizes the spectral radius of the matrix

$$\begin{bmatrix} 1 & 0 \\ -\omega a & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1-\omega & \omega a \\ 0 & 1-\omega \end{bmatrix}$$

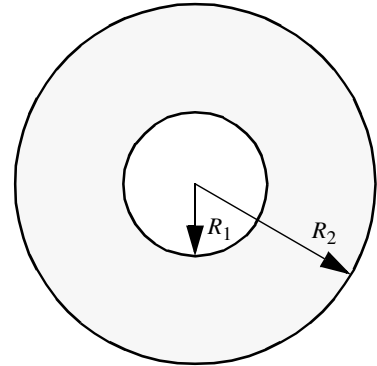
and explain why we would want to do this.

- 2) Consider two grounded, infinitely long coaxial cylinders as shown in the figure. The space between the cylinders is filled with a non-uniformly charged dielectric material, having a variable charge density given by $\rho(r) = \rho_0 r / R_1$ with relative permeability $\epsilon_r = 2$, and $\rho_0 = 10^{-6} \text{ C/m}^2$. The dimensions are: $R_1 = 1 \text{ cm}$, $R_2 = 10 \text{ cm}$.

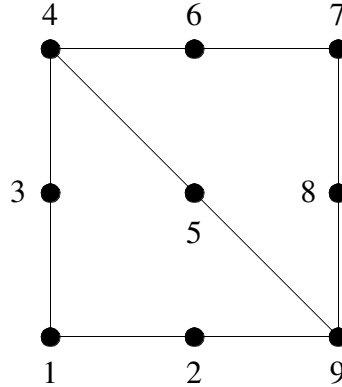
a) Solve the problem analytically by integrating Poisson's equation in cylindrical coordinates.

b) Solve the problem using either the Finite-Element Method or Galerkin's method and at least 3 linear elements.

c) Compare the analytical and numerical solutions at the nodes.



- 3) For the following finite element mesh assemble the global matrix $[S]$ assuming a second order approximation for the solution. Each triangle has one 90° and two 45° angles.



- 4) Show that Yee's algorithm for solving Maxwell's equations can be derived by starting from the integral form of Maxwell's equations instead of from the differential form. Derive all six explicit update equations for a homogeneous lossless medium.
- 5) In deriving the Mur absorbing boundary conditions a differencing operator which takes differences of average values is used. For example, in considering the one-way wave equation:

$$\partial_t u(x, t) - c_0 \partial_x u(x, t) = 0$$

and the grid function given by $u_i^n \equiv u(i\Delta x, n\Delta t)$, the following discretization is used:

$$\frac{[(u_1^{n+1} + u_0^{n+1}) - (u_1^n + u_0^n)]}{\Delta t} - c_0 \frac{[(u_1^{n+1} + u_1^n) - (u_0^{n+1} + u_0^n)]}{\Delta t} = 0$$

at the point $(0, n\Delta t)$. What is the truncation error of this scheme for the above PDE? What is the stability limit in terms of Δx and Δt .